Optimization Fall 2020

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Game Schedule for U.S. National Football League

1. Plain Vanilla Schedule

**- What are the decision variables in this problem?**

The decision variable should be considered as a binary indication of weather home team ℎ would play against away team 𝑎 on week 𝑤w. For example, if 𝑥ℎ,𝑎,𝑤equals to 1, is means that the home team ℎ would play against with away team 𝑎 on week 𝑤w. Otherwise, the game does not happen.

**- What is the objective function? What are the constraints?**

The objective function which minimizes the total travel distance of all teams should be the summed product of all the teams travel distances throughout the 12 weeks. If we assume the travel distance from the origin to destination as 𝑑ℎ,𝑎， the objective function should be defined as

𝑚𝑖𝑛∑ℎ,𝑎,𝑤𝑥ℎ,𝑎,𝑤⋅2𝑑ℎ,𝑎

The constraints for the scheduling, as discussed in the problem statement as follows

1. The season was limited to 12 weeks.
2. Each team would play once per week.
3. All 12 games that a team played would need to be against a different opponent.
4. Each team would play at most six home games (i.e., on their home stadium).

#### - Write down the mathematical formulation of the problem.

𝑚𝑖𝑛∑ℎ,𝑎,𝑤𝑥ℎ,𝑎,𝑤⋅2𝑑ℎ,𝑎

subject to

∑ℎ∑𝑤[𝑥ℎ,𝑎,𝑤+𝑥ℎ,𝑎,𝑤]=12

∑ℎ[𝑥ℎ,𝑎,𝑤+𝑥𝑎,ℎ,𝑤]=1

∑𝑤[𝑥ℎ,𝑎,𝑤+𝑥𝑎,ℎ,𝑤]<=1

∑ℎ∑𝑤𝑥ℎ,𝑎,𝑤<=6

#### Develop your optimization model and save it as “nfl 1.py”.

1. the minimum total distance travelled by all teams

As shown in the results, the best objective is 165908, which is the minimum total travel distance of all the teams.

(ii) illustrate the optimal schedule for Cleveland Browns. How does the home and away game pattern look like for the Cleveland Browns?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Week | Home Team | Away Team | Travel Distance |  |
| 1 | Cleveland Browns | 19 | 1278 | x[14,19,0] : 1278 |
| 2 | 3 | Cleveland Browns | 340 | x[3,14,1] : 340 |
| 3 | Cleveland Browns | 4 | 1100 | x[14,4,2] : 1100 |
| 4 | 16 | Cleveland Browns | 630 | x[16,14,3] : 630 |
| 5 | 2 | Cleveland Browns | 688 | x[2,14,4] : 688 |
| 6 | 13 | Cleveland Browns | 500 | x[13,14,5] : 500 |
| 7 | 5 | Cleveland Browns | 1504 | x[5,14,6] : 1504 |
| 8 | Cleveland Browns | 20 | 920 | x[14,20,7] : 920 |
| 9 | 12 | Cleveland Browns | 370 | x[12,14,8] : 370 |
| 10 | Cleveland Browns | 11 | 748 | x[14,11,9] : 748 |
| 11 | Cleveland Browns | 7 | 912 | x[14,7,10] : 912 |
| 12 | Cleveland Browns | 21 | 266 | x[14,21,11] : 266 |

Cleveland Browns hosts 6 home games and travel 6 times for away games. Its total travel distance for away games is 5224 miles and its opponents' travel distance to Cleveland Browns home game is 4032 miles.

1. Game Pattern and Bye Week Constraints

The NFL schedulers asked whether it would be possible to adjust the plain vanilla schedule (which followed the four main rules) to obtain a schedule in which no team would play more than two consecutive games at home or two consecutive games away. The logic behind these game pattern constraints was to provide a fairer schedule for the teams in terms of them having similar home/away game patterns.

**-What is the mathematical formulation for the game pattern constraints?**

In order to have no team play no more than two consecutive games at home or away, we need to have another two binary decision variables y(h,w) and z(h,w).

y(h,w) denotes to weather home team h plays at home in weeks w and w+1. If y(h,w) =1, it means that home team h plays at home on weeks w and w+1, y(h,w)= 0 otherwise.

z(h,w) denotes to weather home team h plays away in weeks w and w+1. If z(h,w) =1, it means that home team h plays away on weeks w and w+1, z(h,w)= 0 otherwise.

**-What are the implications of the game pattern constraints?**

If y(h,w) and z(h,w) equals to 1, it means the home team h played two consecutive games at home or away. And implies another extra constraint.

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**-What happened to the minimum total distance travelled by all teams and the optimal schedule for Cleveland Browns?**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Week | Home Team | Away Team | Travel Distance |  |
| 1 | Cleveland Browns🡪19 | 19🡪 Cleveland Browns | 1278 | x[19,14,0] : 1278 |
| 2 | 3 | Cleveland Browns | 340 | x[3,14,1] : 340 |
| 3 | Cleveland Browns | 4 | 1100 | x[14,4,2] : 1100 |
| 4 | 16🡪21 | Cleveland Browns | 266 | x[21,14,3] : 266 |
| 5 | 2🡪 Cleveland Browns | Cleveland Browns🡪2 | 688 | x[14,2,4] : 688 |
| 6 | 13🡪7 | Cleveland Browns | 912 | x[7,14,5] : 912 |
| 7 | 5🡪 Cleveland Browns | Cleveland Browns🡪11 | 748 | x[14,11,6] : 748 |
| 8 | Cleveland Browns🡪16 | 20🡪 Cleveland Browns | 630 | x[16,14,7] : 630 |
| 9 | 12🡪 Cleveland Browns | Cleveland Browns🡪12 | 370 | x[14,12,8] : 370 |
| 10 | Cleveland Browns🡪20 | 11🡪 Cleveland Browns | 920 | x[20,14,9] : 920 |
| 11 | Cleveland Browns | 7 🡪5 | 1504 | x[14,5,10] : 1504 |
| 12 | Cleveland Browns | 21🡪13 | 500 | x[14,13,11] : 500 |

The minimum total travel distance of all the teams is still 165908.

To further reduce the fatigue caused by the competition and travelling, the NFL schedulers asked to incorporate in the schedule a week when there are no games. This one-week long break is also referred as the bye week. Suppose that the schedulers requested the bye week to be assigned on week 9, which means that there are no games at week 9, and that the whole season is extended by a week up to 13 weeks.

**-What are the required changes for the mathematical formulation to incorporate also the bye week constraint?**

1. The total number of weeks is now adding up to 13
2. For week 9, there is no game happen

 when  = 9